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PLANE CONTACT PROBLEM OF INTERACTION BETWEEN A RIGID HEAT-CONDUCTING WEDGE AND AN ELASTIC LAYER UNDER NONSTATIONARY FRICTION HEAT RELEASE

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We consider a solution to the new plane problem of contact interaction with an elastic layer of a rigid, heat-conducting cross-sectional die in the form of a bounded or unbounded wedge. Using the apparatus of integral transformations, we have obtained an exact solution of the heat-conduction equations for the die and the layer, which permits reducing the problem stated to a system of integral equations whose structure is determined by the kind of thermophysical conditions on the interaction surface. This enables us to make the mathematical statement of the problem more realistic in terms of the distribution of thermoelastic stresses and adequately estimate the influence of thermal fields on the value of the contact pressure and the character of its distribution.

Introduction. The classical, at present, formulation of thermoelastic contact problems with friction heat release presupposes the interaction of a bounded, rigid, heat-insulated die with the surface of an elastic half-space or layer [1–3]. But if a heat-conducting die is considered, then this body is modeled by a half-plane or a half-space, which makes it possible to use, in calculating the temperature fields, differential heat-conduction equations for semibounded bodies and, accordingly, the mathematical apparatus of integral transformations or construction of Green functions [4, 5]. Evidently, these model simplifications introduce significant corrections into the obtained distributions of contact stresses and temperature fields. Moreover, problems on the influence of friction heat release on the mechanism of interaction with the base of a die in the form of a wedge are disregarded by researchers, whereas contact problems of the elasticity theory on pressing a rigid wedge into a half-space were considered in monographs [6, 7]. At the same time, this class of problems should not be ignored, since the body in the form of a wedge models the action of a cutting tool and the investigation of the thermomechanical processes that take place in machine working of a material is necessary for developing effective technologies in machine building. Therefore, complication of models with the aim of making them more realistic in terms of real tribosystems is the goal of mathematical problems of tribology.

The present paper considers a new quasi-static contact problem for a tribosystem consisting of an elastic layer of finite length h fixed on a base on whose surface a heat-conducting die in the form of a wedge moves. In so doing, the heat release due to the action of forces obeying the Amonton (Coulomb) law is taken into account. The formulation of the problem takes into account the die geometry and, as a consequence, with the help of the apparatus of integral transformations, the exact solution of the nonstationary heat-conduction equation for the wedge-shaped region and the exact solution of the nonstationary heat-conduction equations and quasi-static thermal elasticity for the layer are constructed. The problem has been reduced to a system of integral equations with time-varying integration boundaries, whose structure depends on the kind of thermophysical conditions on the interaction surface. The numerical solution of integral equations is constructed on the basis of the known algorithm [8, 9], which makes it possible to analyze the effect of the change in the contact pressure and in the value of the interaction area with changing-with-time pressing force and velocity of motion.

Mathematical Formulation of the Problem. Let a rigid, wedge-shaped, in the cross-section, die with an apex angle $|\varphi| \le \alpha_0$ unbounded ($0 \le \rho < \infty$) or bounded by a circular arc $\rho = b$ be pressed by force $P(\tau)$, referred to a unit length, to an elastic layer of thickness h rigidly fixed on the base (Fig. 1). It is assumed that the die in the direction

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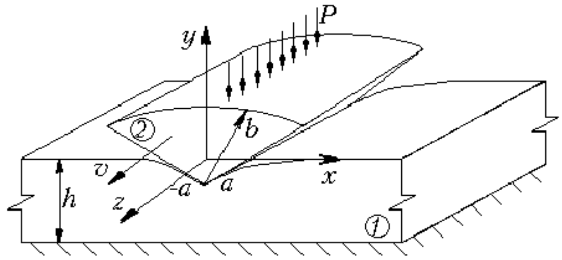

Fig. 1. Scheme of the problem of contact interaction between a rigid, heat-conducting cross-sectional wedge-shaped die and an elastic layer.

of the z-axis moves on the layer surface at a low velocity $v(\tau)$. In so doing, the changes with time in the pressing force and velocity of motion are such that the possible dynamic effects can be neglected and the problem is considered in the quasistatic formulation. As a consequence of the action of forces τ_{yz} arising on the contact surfaces and obeying the Amonton (Coulomb) law $\tau_{yz} = f\sigma_y$, in the contact plane nonstationary heat release occurs. The contact region between the die and the layer is described by the inequalities $|x| \le a(\tau)$; $|z| < \infty$, where the half-width of the interaction area $a(\tau)$ can vary with time, which is due to the nonstationary character of the heat release. Since the die is heat-conducting, the heat generated on the contact is distributed between the bodies and causes their heating. That is why the contact surface of the layer bulges and, as a consequence, the half-width of the interaction area $a(\tau)$ changes.

Between the lower surface of the layer and the base, whose temperature is equal to zero, as well as between the upper plane outside the contact area and the environment with a zero temperature heat exchange by the Newton law is supposed. Outside the contact area $0 \le \rho \le a(\tau)/\sin(\alpha_0)$, the surface of the wedge is assumed to be heat-insulated, which is required by the conditions of using integral transformations — by the possibility of separating variables in the initial-boundary-value problem for the heat-conduction equation. Moreover, the influence of tangent stresses τ_{yx} in the contact area is neglected.

Under the above assumptions realizing a plane deformation in the layer the problem is reduced to the construction of solutions of a system including the differential heat conduction equations for the layer in the Cartesian coordinate system

$$\partial_x^2 T_1 + \partial_y^2 T_1 = k_1^{-1} \partial_\tau T_1 \tag{1}$$

and for the wedge-shaped, in plan, die in the polar coordinates

$$\partial_\rho^2 T_2 + \rho^{-1} \partial_\rho T_2 + \rho^{-2} \partial_\varphi^2 T_2 = k_2^{-1} \partial_\tau T_2 \,, \tag{2}$$

as well as for the thermoelasticity (only for the layer)

$$\begin{gathered} (1-2\nu)\left(\partial_x^2 u_x + \partial_y^2 u_x\right) + \partial_x \left(\partial_x u_x + \partial_y u_y\right) = 2\alpha\,(1+\nu)\,\partial_x T_1 \,, \\ (1-2\nu)\left(\partial_x^2 u_y + \partial_y^2 u_y\right) + \partial_y \left(\partial_x u_x + \partial_y u_y\right) = 2\alpha\,(1+\nu)\,\partial_y T_1 \end{gathered} \tag{3}$$

under the initial

$$T_1(x, y, 0) = 0 \,, \quad T_2(\rho, \varphi, 0) = 0 \,, \tag{4}$$

boundary and contact conditions:

$$y = -h: \;\; \partial_y T_1 = \gamma_1 T_1 \,, \;\; u_x = 0 \,, \;\; u_y = 0 \,; \tag{5}$$

$$y=0: \quad |x| \le a(\tau): \quad u_y = -\delta(\tau) + \mathrm{ctan}(\alpha_0)\,|x|, \quad \tau_{yx}=0; \tag{6}$$

$$\lambda_1 \partial_y T_1 - \lambda_2 \partial_y T_2 = fv(\tau)\, p(x,\tau), \quad \lambda_1 \partial_y T_1 + \lambda_2 \partial_y T_2 + h_0 (T_1 - T_2) = 0; \tag{7}$$

$$|x| > a(\tau): \quad \partial_y T_1 = -\gamma_0 T_1, \quad \sigma_y = 0; \quad \tau_{yx} = 0; \tag{8}$$

$$\rho = b, \quad |\varphi| \le \alpha_0: \quad \partial_\rho T_2 = -\beta_0 T_2, \quad a(\tau)/\sin(\alpha_0) < \rho \le b, \quad \varphi = \pm\alpha_0: \quad \rho^{-1}\partial_\varphi T_2 = 0; \tag{9}$$

$$\rho \to \infty, \quad |\varphi| \le \alpha_0: \quad T_2 \to 0; \quad a(\tau)/\sin(\alpha_0) < \rho < \infty; \quad \varphi = \pm\alpha_0: \quad \rho^{-1}\partial_\varphi T_2 = 0. \tag{10}$$

The stresses in the layer are defined by the formulas

$$\sigma_x = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(\partial_x u_x + \frac{\nu}{1-\nu}\partial_y u_y - \alpha\frac{1+\nu}{1-\nu}T_1\right),$$

$$\sigma_y = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(\partial_y u_y + \frac{\nu}{1-\nu}\partial_x u_x - \alpha\frac{1+\nu}{1-\nu}T_1\right), \quad \tau_{yx} = \frac{E}{2(1+\nu)}\left(\partial_y u_x + \partial_x u_y\right). \tag{11}$$

The system of differential equations and boundary conditions is closed by the expressions of boundedness of the contact pressure $p(\pm a(\tau), \tau) = 0$ valid for an unbounded or a high wedge ($b > a(\tau)/\sin(\alpha_0)$), as well as by the equilibrium relation of the die as a rigid body

$$\int_{-a(\tau)}^{a(\tau)} p(x,\tau)\,dx = P(\tau). \tag{12}$$

The value of the index $j = 1$ corresponds to the layer and $j = 2$ — to the rigid die.

The above formulation of the thermoelastic problem (1)–(12) requires some explanation.

1. In giving the contact conditions for the layer, the boundary conditions are carried onto the undeformed surface, as is commonly done in problems of the linear theory of elasticity. Moreover, it should be noted that for the in-plan wedge-shaped die the half-width of the contact area is primordially a variable quantity, the change in which is equally influenced by both mechanical (e.g., an increase in the pressing force) and thermophysical factors (bulging of the layer surface under friction heating).

2. The boundary conditions (9) are used for the interaction of a bounded wedge with the layer, where the first relation in (9) describes the heat exchange from the surface $\rho = b$ of the wedge by the Newton law with a zero-temperature medium, and the second relation describes the condition of heat insulation of the wedge surface outside the interaction area. The first relation in the boundary condition (10) describes the temperature decay in the unbounded wedge at infinity, and the second one describes the heat insulation of the unbounded wedge surface outside the contact area.

3. It is more convenient to calculate the temperature field in the wedge in the polar system of coordinates. Therefore, to match the temperature function of the die under the thermophysical conditions (7) — the condition of heat release and nonideal thermal contact — the relations $\rho = \sqrt{x^2 + y^2}$ and $\varphi = \arctan(x/y)$ will be used.

Construction of the Solution. To obtain the solution, let us divide the initial boundary problem of thermoelasticity given by relations (1)–(12) into two independent problems — construction of the solution to the nonstationary heat conduction problem and its corresponding quasi-static heat elasticity problem for the layer and differentiation of the nonstationary heat conduction equation at mixed boundary conditions for the in-plan wedge-shaped die. To this end, let us introduce into consideration two unknown functions which are a linear combination of the temperature and the heat flow on the contact surface:

$$f_1(x,\tau)=\left[\partial_y T_1(x,0,\tau)+\gamma_0 T_1(x,0,\tau)\right] S\left(a(\tau)-|x|\right),\quad f_2(x,\tau)=-\partial_y T_2(x,0,\tau)\, S\left(a(\tau)-|x|\right), \tag{13}$$

where $S(x)$ is the Heaviside function [10]. Then the layer temperature is determined from the solution of the differential equation (1) at the initial condition from (4) and the boundary condition

$$\partial_y T_1(x,-h,\tau)=\gamma_1 T_1(x,-h,\tau),\quad \partial_y T_1(x,0,\tau)=f_1(x,\tau)\, S\left(a(\tau)-|x|\right)-\gamma_0 T_1(x,0,\tau),$$

where the full scheme of constructing the integral representation of the layer temperature in terms of the sought function $f_1(x,\tau)$ and the corresponding calculations are given in [8, 9].

We seek a solution of the system of differential thermoelasticity equations (3) in the form of a sum of the general solution of the homogeneous system and a particular solution of the inhomogeneous system where the thermoelastic potential of displacements [11] $F(x, y, \tau)$, which is a particular solution of the differential equation $(\partial_x^2+\partial_y^2)F(x,y,\tau)=\alpha(1+\nu)(1-\nu)^{-1}T_1(x,y,\tau)$, is used. Applying to Eq. (3) and the boundary conditions

$$u_x(x,-h,\tau)=0,\quad u_y(x,-h,\tau)=0,\quad \sigma_y(x,0,\tau)=-p(x,\tau)\, S\left(a(\tau)-|x|\right),\quad \tau_{yx}(x,0,\tau)=0,$$

the integral Fourier transform on the x coordinate, solving in transforms the obtained systems of ordinary differential equations, and transforming the expressions for the Fourier transforms of thermoelastic displacements and stresses, we will have a solution of the quasistatic thermoelastic problem — integral representations for the sought components of the temperature and stressed–strained states of the layer for the sought function $f_1(x,\tau)$ and contact pressure $p(x,\tau)$.

Here we will give only the formulas for the contact temperature and normal displacements of the surface $y = 0$ of the elastic layer in dimensionless form. To this end, we assign the linear dimensions of the body to the layer thickness h, the stresses to the value of P_0h^{-1}, and the temperature to the combination of parameters $\alpha Eh(2P_0(1-\nu))^{-1}$, and, as a result, we obtain

$$T_1(x,y,\mathrm{Fo})=\frac{1}{\pi}\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_1(t,\eta)\,\Phi_1(t-x,y,\mathrm{Fo}-\eta)\,dt\,d\eta,$$

$$u_y(x,0,\mathrm{Fo})=-\frac{2P_0(1-\nu^2)}{Eh}\frac{1}{\pi}\left[\int_{-a(\tau)}^{a(\tau)} p(t,\mathrm{Fo})\,\Delta(t-x)\,dt-\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_1(t,\eta)\,H(t-x,\mathrm{Fo}-\eta)\,dt\,d\eta\right], \tag{14}$$

when

$$\Phi_1(x,y,\mathrm{Fo})=\int_0^\infty \frac{\xi\cosh(\xi(1+y))+\mathrm{Bi}_{1,1}\sinh(\xi(1+y))}{\xi(\xi\sinh(\xi)+\mathrm{Bi}_{1,1}\cosh(\xi))+\mathrm{Bi}_{0,1}(\xi\cosh(\xi)+\mathrm{Bi}_{1,1}\sinh(\xi))}\cos(\xi x)\,d\xi$$

$$-\frac{\pi}{2}\sum_{m=1}^{\infty}\frac{\mu_m\cos(\mu_m y)-\mathrm{Bi}_{0,1}\sin(\mu_m y)}{(\mu_m^2+\mathrm{Bi}_{0,1}^2)\left(1+\mathrm{Bi}_{1,1}(\mu_m^2+\mathrm{Bi}_{1,1}^2)^{-1}\right)+\mathrm{Bi}_{0,1}}\sum_{k=1}^{2}\exp\left((-1)^k\mu_m x\right)\mathrm{erfc}\left(\mu_m\sqrt{\mathrm{Fo}}+(-1)^k\frac{x}{2\sqrt{\mathrm{Fo}}}\right),$$

$$\Delta(x)=\int_0^\infty \xi^{-1}\frac{(3-4\nu)\cosh(\xi)\sinh(\xi)-\xi}{\xi^2-(1-2\nu)^2\sinh^2(\xi)+4(1-\nu)^2\cosh^2(\xi)}\cos(\xi x)\,d\xi,$$

$$H(x, \mathrm{Fo}) = \int_0^{\infty} \Bigg(4 \frac{2(1-\nu)(\xi \sinh(\xi) - \mathrm{Bi}_{1,1} \cosh(\xi)) - \xi (\sinh(\xi) + \xi \cosh(\xi) + \mathrm{Bi}_{1,1} \sinh(\xi))}{\xi^2 - (1-2\nu)^2 \sinh^2(\xi) + 4(1-\nu)^2 \cosh^2(\xi)}$$

$$\times \sum_{m=1}^{\infty} \frac{\mu_m}{(\mu_m^2 + \xi^2)^2} \frac{(\mu_m \cos(\mu_m) + \mathrm{Bi}_{0,1} \sin(\mu_m)) \exp(-(\xi^2 + \mu_m^2)\mathrm{Fo})}{(\mu_m^2 + \mathrm{Bi}_{0,1}^2)(1 + \mathrm{Bi}_{1,1}(\mu_m^2 + \mathrm{Bi}_{1,1}^2)^{-1}) + \mathrm{Bi}_{0,1}}$$

$$+ \frac{1}{\xi} \frac{(\xi^2 + (3-4\nu)\sinh^2(\xi))(\xi \sinh(\xi) + \mathrm{Bi}_{1,1} \cosh(\xi)) + 2(1-\nu)\xi(\xi \cosh(\xi) - \mathrm{Bi}_{1,1} \sinh(\xi) + \sinh(\xi))}{(\xi(\xi \sinh(\xi) + \mathrm{Bi}_{1,1} \cosh(\xi)) + \mathrm{Bi}_{0,1}(\xi \cosh(\xi) + \mathrm{Bi}_{1,1} \sinh(\xi)))(\xi^2 - (1-2\nu)^2 \sinh^2(\xi) + 4(1-\nu)^2 \cosh^2(\xi))}$$

$$+ 4 \frac{\xi^2 - 2(1-\nu)\mathrm{Bi}_{0,1} - (3-4\nu)\sinh(\xi)(\xi(\cosh(\xi) + \mathrm{Bi}_{0,1} \sinh(\xi))}{\xi^2 - (1-2\nu)^2 \sinh^2(\xi) + 4(1-\nu)^2 \cosh^2(\xi)}$$

$$\times \sum_{m=1}^{\infty} \frac{\mu_m^2}{(\mu_m^2 + \xi^2)^2} \frac{\exp(-(\xi^2 + \mu_m^2)\mathrm{Fo})}{(\mu_m^2 + \mathrm{Bi}_{0,1}^2)(1 + \mathrm{Bi}_{1,1}(\mu_m^2 + \mathrm{Bi}_{1,1}^2)^{-1}) + \mathrm{Bi}_{0,1}} \Bigg) \cos(\xi x)\, d\xi,$$

$\mathrm{Fo} = \tau k_1 h^{-2}$, $\mathrm{Bi}_{0,1} = \gamma_0 h$ and $\mathrm{Bi}_{1,1} = \gamma_1 h$ are Fourier and Biot criteria [12], erfc (x) is the error function [10], μ_m stands for positive roots of the transcendental equation of the Sturm–Liouville problem:

$$\mu_m (\mu_m \sin(\mu_m) - \mathrm{Bi}_{1,1} \cos(\mu_m)) - \mathrm{Bi}_{0,1} (\mu_m \cos(\mu_m) + \mathrm{Bi}_{1,1} \sin(\mu_m)) = 0 .$$

In the above formulas for the temperature and normal displacements, no new variables were introduced for the x, y coordinates assigned to the layer thickness h and the half-widths of the contact area $a(\tau)$, as well as for the contact pressure function $p(x, \mathrm{Fo})$ assigned to the combination of parameters $P_0 h^{-1}$.

The nonstationary temperature field for the in-plan wedge-shaped region is obtained from the solution of the differential heat conduction equation in the polar coordinate system (2) at the initial condition from (4) and the boundary conditions

$$\rho^{-1}\partial_\varphi T_2(\rho, \alpha_0, \tau) = f_2(\rho \sin(\alpha_0), \tau)\, S(a(\tau)/\sin(\alpha_0) - \rho)/\sin(\alpha_0),$$

$$\rho^{-1}\partial_\varphi T_2(\rho, -\alpha_0, \tau) = -f_2(\rho \sin(\alpha_0), \tau)\, S(a(\tau)/\sin(\alpha_0) - \rho)/\sin(\alpha_0), \tag{15}$$

$$\rho = b, \quad |\varphi| \le \alpha_0: \ \partial_\rho T_2 = -\beta_0 T_2 \quad \text{(for the bounded wedge);}$$

$$\rho \to \infty, \quad |\varphi| \le \alpha_0: \ T_2 \to 0 \quad \text{(for the unbounded wedge).}$$

The first two relations in (15) are the boundary conditions on the side surface of the wedge, where it is taken into account that from the physical considerations of the formulation of the problem the function $f_2(x, \tau)$ is pair on the x coordinate. The other two relations in (15) define, respectively, the condition of heat transfer from the surface $\rho = b$ of the bounded wedge and the condition of temperature decay at infinity for the unbounded wedge-shaped region.

The construction of the solution of the heat conduction equation (2), (4), and (15) consists of using two integral transforms on the angular and radial coordinates. For both the finite and the infinite wedge, we first use the finite integral Fourier transform on the angular coordinate φ with a kernel

$$K_1(\varphi, n) = \begin{cases} 1/\sqrt{2\alpha_0}, & n = 0; \\ \cos(\pi n \varphi/\alpha_0)/\sqrt{\alpha_0}, & n \ge 1. \end{cases}$$

Then on the radial coordinate ρ for the bounded wedge we use the finite integral Hankel transform [13] with a kernel

$$K_2(\rho, \mu_{m,n}) = \frac{\sqrt{2}}{b} \frac{1}{\sqrt{1 + \dfrac{\beta_0^2}{\mu_{m,n}^2} - \dfrac{\pi^2 n^2}{\alpha_0^2 b^2 \mu_{m,n}^2}}} \frac{J_{\frac{\pi n}{\alpha_0}}(\mu_{m,n}\rho)}{J_{\frac{\pi n}{\alpha_0}}(\mu_{m,n} b)},$$

and for the unbounded wedge — the integral Hankel transform [13], where $J_n(x)$ is a Bessel function of the first kind of order n [10]; $\mu_{m,n}$ stands for positive roots of the transcendental equation of the Sturm–Liouville problem

$$(\pi n/\alpha_0 + b\beta_0)\, J_{\frac{\pi n}{\alpha_0}}(\mu_{m,n} b) - \mu_{m,n} b J_{\frac{\pi n}{\alpha_0}+1}(\mu_{m,n} b) = 0,$$

where at $n = 0$ the first eigenvalue of $\mu \neq 0$, and at $n \geq 1$ $\mu = 0$ is a root. The application of the integral transforms permits obtaining, for determining the Fourier–Hankel transform of the wedge temperature, a Cauchy problem, whose solution and inversion of the double integral Fourier–Hankel transform enable us to write the expression for the temperature of the wedge-shaped region:

$$T_2(\rho, \varphi, \tau) = \partial_\tau \int_0^\tau \int_0^{a(\eta)/\sin(\alpha_0)} f_2(t \sin(\alpha_0), \eta)\, \Phi_2(\rho, t, \varphi, \tau - \eta)\, dt\, d\eta, \tag{16}$$

where for the bounded wedge the kernel of the integral representation is of the form

$$\Phi_2(\rho, t, \varphi, \tau) = \frac{1}{b^2 \alpha_0 \sin(\alpha_0)} \sum_{m=1}^{\infty} \frac{J_0(\mu_{m,0}\rho)\, J_0(\mu_{m,0} t)}{\left(\beta_0^2 + \mu_{m,0}^2\right) J_0^2(\mu_{m,0} b)} \left(1 - \exp\left(-\mu_{m,0}^2 k_2 \tau\right)\right) + \frac{2}{b^2 \alpha_0 \sin(\alpha_0)}$$

$$\times \sum_{n=1}^{\infty} (-1)^n \cos(\pi n \varphi/\alpha_0) \sum_{m=0}^{\infty} \frac{J_{\frac{\pi n}{\alpha_0}}(\mu_{m,n}\rho)\, J_{\frac{\pi n}{\alpha_0}}(\mu_{m,n} t)}{\left(\left(\beta_0^2 + \mu_{m,n}^2\right) - b^{-2} (\pi n/\alpha_0)^2\right) J_{\frac{\pi n}{\alpha_0}}^2(\mu_{m,n} b)} \left(1 - \exp\left(-\mu_{m,n}^2 k_2 \tau\right)\right),$$

and in the case of the unbounded wedge-shaped region

$$\Phi_2(\rho, t, \varphi, \tau) = \frac{1}{\alpha_0 \sin(\alpha_0)} \int_0^\infty \zeta^{-1} J_0(\zeta\rho)\, J_0(\zeta t) \left(1 - \exp\left(-\zeta^2 k_2 \tau\right)\right) d\zeta + \frac{2}{\alpha_0 \sin(\alpha_0)}$$

$$\times \sum_{n=1}^{\infty} (-1)^n \cos(\pi n \varphi/\alpha_0) \int_0^\infty \zeta^{-1} J_{\frac{\pi n}{\alpha_0}}(\zeta\rho)\, J_{\frac{\pi n}{\alpha_0}}(\zeta t) \left(1 - \exp\left(-\zeta^2 k_2 \tau\right)\right) d\zeta.$$

The final solution of the problem is constructed in the Cartesian system of coordinates $x0y$. Therefore, using the relations $\rho = \sqrt{x^2 + y^2}$ and $\varphi = \arctan(x/y)$, we give the corresponding expressions for the wedge temperature, as was done for the elastic layer, in this coordinate system in dimensionless form

$$T_2(x, y, \mathrm{Fo}) = \partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} f_2(t, \eta)\, \Phi_2(x, t, y, \mathrm{Fo} - \eta)\, dt\, d\eta, \tag{17}$$

where for the bounded wedge the kernel of the integral representation has the form

$$\Phi_2(x,t,y,\mathrm{Fo}) = \frac{1}{b^2\alpha_0\sin^2(\alpha_0)}\sum_{m=1}^{\infty}\frac{J_0\left(\mu_{m,0}\sqrt{x^2+y^2}\right)J_0\left(\mu_{m,0}\,|t|/\sin(\alpha_0)\right)}{\left(\mathrm{Bi}_{0,2}^2+\mu_{m,0}^2\right)J_0^2(\mu_{m,0}b)}$$

$$\times\left(1-\exp\left(-\mu_{m,0}^2k\mathrm{Fo}\right)\right)+\frac{2}{b^2\alpha_0\sin^2(\alpha_0)}\sum_{n=1}^{\infty}(-1)^n\cos\left(\pi n\arctan(x/y)/\alpha_0\right)$$

$$\times\sum_{m=0}^{\infty}\frac{J_{\frac{\pi n}{\alpha_0}}\left(\mu_{m,n}\sqrt{x^2+y^2}\right)J_{\frac{\pi n}{\alpha_0}}\left(\mu_{m,n}\,|t|/\sin(\alpha_0)\right)}{\left(\left(\mathrm{Bi}_{0,2}^2+\mu_{m,n}^2\right)-b^{-2}(\pi n/\alpha_0)^2\right)J_{\frac{\pi n}{\alpha_0}}^2(\mu_{m,n}b)}\left(1-\exp\left(-\mu_{m,n}^2k\mathrm{Fo}\right)\right),$$

and for the unbounded wedge-shaped region

$$\Phi_2(x,t,y,\mathrm{Fo}) = \frac{1}{\alpha_0\sin^2(\alpha_0)}\int_0^{\infty}\zeta^{-1}J_0\left(\zeta\sqrt{x^2+y^2}\right)J_0\left(\zeta\,|t|/\sin(\alpha_0)\right)\left(1-\exp\left(-\zeta^2k\mathrm{Fo}\right)\right)d\zeta$$

$$+\frac{2}{\alpha_0\sin^2(\alpha_0)}\sum_{n=1}^{\infty}(-1)^n\cos\left(\pi n\arctan(x/y)/\alpha_0\right)$$

$$\times\int_0^{\infty}\zeta^{-1}J_{\frac{\pi n}{\alpha_0}}\left(\zeta\sqrt{x^2+y^2}\right)J_{\frac{\pi n}{\alpha_0}}\left(\zeta\,|t|/\sin(\alpha_0)\right)\left(1-\exp(-\zeta^2k\mathrm{Fo})\right)d\zeta,$$

where $k = k_2/k_1$; $\mathrm{Bi}_{0,2} = \beta_0 h$. We did not introduce into the formulas of the wedge temperature new variables for the arc radius of the bounded wedge b assigned to the layer thickness h and eigenvalues of the Sturm–Liouville problem $\mu_{m,n}$ multiplied by h, i.e, they are determined from the transcendental equation

$$\left(\pi n/\alpha_0+b\mathrm{Bi}_{0,2}\right)J_{\frac{\pi n}{\alpha_0}}(\mu_{m,n}b)-\mu_{m,n}bJ_{\frac{\pi n}{\alpha_0}+1}(\mu_{m,n}b)=0\,.$$

Having satisfied the last three conditions of the problem, namely the kinematic contact condition from (6) and the relations of heat conduction and nonideal thermal contact (7), we obtain a system of integral equations of the problem stated whose structure is determined by the kind of thermophysical contact conditions. In particular, at a nonideal thermal contact we choose for the sought functions by varying $|x| \le a(\mathrm{Fo})$, the contact pressure and temperature of the contact segment $T_{0,j}(x, \mathrm{Fo}) = T_j(x, 0, \mathrm{Fo})$ determined in terms of the function $f_j(x, \mathrm{Fo})$ from the thermophysical contact conditions (7):

$$f_1(x,\mathrm{Fo}) = \frac{1}{2}\chi v_*(\mathrm{Fo})\,p(x,\mathrm{Fo})+\left(\mathrm{Bi}_{0,1}-\frac{h_*}{2}\right)T_{0,1}(x,\mathrm{Fo})+\frac{h_*}{2}T_{0,2}(x,\mathrm{Fo})\,;$$

(18)

$$f_2(x,\mathrm{Fo}) = \frac{1}{2\Lambda}\chi v_*(\mathrm{Fo})\,p(x,\mathrm{Fo})+\frac{h_*}{2\Lambda}T_{0,1}(x,\mathrm{Fo})-\frac{h_*}{2\Lambda}T_{0,2}(x,\mathrm{Fo})\,,$$

where $\Lambda = \lambda_2/\lambda_1$; $h_* = h_0 h \lambda_1^{-1}$; $\chi = \dfrac{\alpha E h f v_0}{2\lambda_1(1-\nu)}$. Substituting expressions (18) into the temperature formulas (14) and (17), we get new integral representations for the temperature of the bodies

$$T_1(x, y, \mathrm{Fo}) = \frac{\chi}{2\pi}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} v_*(\eta)\, p(t, \eta)\, \Phi_1(t-x, y, \mathrm{Fo}-\eta)\, dt\, d\eta + \frac{1}{\pi}\left(\mathrm{Bi}_{0,1} - \frac{h_*}{2}\right)$$

$$\times \partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} T_{0,1}(t, \eta)\, \Phi_1(t-x, y, \mathrm{Fo}-\eta)\, dt\, d\eta + \frac{1}{\pi}\frac{h_*}{2}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} T_{0,2}(t, \eta)\, \Phi_1(t-x, y, \mathrm{Fo}-\eta)\, dt\, d\eta\,; \qquad (19)$$

$$T_2(x, y, \mathrm{Fo}) = \frac{\chi}{2\Lambda}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} v_*(\eta)\, p(t, \eta)\, \Phi_2(x, t, y, \mathrm{Fo}-\eta)\, dt\, d\eta$$

$$+ \frac{h_*}{2\Lambda}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} T_{0,1}(t, \eta)\, \Phi_2(x, t, y, \mathrm{Fo}-\eta)\, dt\, d\eta - \frac{h_*}{2\Lambda}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} T_{0,2}(t, \eta)\, \Phi_2(x, t, y, \mathrm{Fo}-\eta)\, dt\, d\eta\,, \qquad (20)$$

which, in the contact segment $|x| \le a(\mathrm{Fo})$, $y = 0$, give two integral equations for determining the unknown $T_{0,j}(x, \mathrm{Fo})$. Together with the relation

$$\frac{1}{\pi}\int_{-a(\mathrm{Fo})}^{a(\mathrm{Fo})} p(t, \mathrm{Fo})\, \Delta(t-x)\, dt - \frac{\chi}{2\pi}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} v_*(\eta)\, p(t, \eta)\, H(t-x, \mathrm{Fo}-\eta)\, dt\, d\eta$$

$$-\frac{1}{\pi}\left(\mathrm{Bi}_{0,1} - \frac{h_*}{2}\right)\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} T_{0,1}(t, \eta)\, H(t-x, \mathrm{Fo}-\eta)\, dt\, d\eta$$

$$-\frac{1}{\pi}\frac{h_*}{2}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} T_{0,2}(t, \eta)\, H(t-x, \mathrm{Fo}-\eta)\, dt\, d\eta = \delta_*(\mathrm{Fo}) - A_* a(\mathrm{Fo})\, |x|\,, \qquad (21)$$

where $\delta_* = \delta E (2P_0(1-\nu^2))^{-1}$ and $A_* = \mathrm{ctan}\,(\alpha_0) E h (2P_0(1-\nu^2))^{-1}$, we arrive at a complete system of integral equations of the problem stated.

At an ideal thermal contact ($h_0 \to \infty$) we choose for the sought functions, by varying $|x| \le a(\mathrm{Fo})$, $f_{0,j}(x, \mathrm{Fo})$ and the temperature of the contact segment $T_1(x, 0, \mathrm{Fo}) = T_2(x, 0, \mathrm{Fo}) = T(x, \mathrm{Fo})$ related to the contact pressure and the functions f_j by the following relations:

$$f_1(x, \mathrm{Fo}) = f_{0,1}(x, \mathrm{Fo}) + \mathrm{Bi}_{0,1} T(x, \mathrm{Fo})\,, \quad f_2(x, \mathrm{Fo}) = f_{0,2}(x, \mathrm{Fo})\,,$$

$$f_{0,1}(x, \mathrm{Fo}) + \Lambda f_{0,2}(x, \mathrm{Fo}) = \chi v_*(\mathrm{Fo})\, p(x, \mathrm{Fo})\,. \qquad (22)$$

In this case, the problem is reduced to the system of integral equations

$$T(x, \mathrm{Fo}) = \frac{1}{\pi}\partial_{\mathrm{Fo}} \int_0^{\mathrm{Fo}} \int_{-a(\eta)}^{a(\eta)} f_{0,1}(t, \eta)\, \Phi_1(t-x, \mathrm{Fo}-\eta)\, dt\, d\eta$$

$$+\frac{1}{\pi}\,\mathrm{Bi}_{0,1}\,\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} T(t,\eta)\,\Phi_1(t-x,0,\mathrm{Fo}-\eta)\,dt\,d\eta\,, \tag{23}$$

$$T(x,\mathrm{Fo})=\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_{0,2}(t,\eta)\,\Phi_2(x,t,0,\mathrm{Fo}-\eta)\,dt\,d\eta\,, \tag{24}$$

$$\frac{1}{\pi}\int_{-a(\mathrm{Fo})}^{a(\mathrm{Fo})}\big(f_{0,1}(t,\mathrm{Fo})+\Lambda f_{0,2}(t,\mathrm{Fo})\big)\,\Delta(t-x)\,dt-\frac{1}{\pi}\,\chi v_*(\mathrm{Fo})$$

$$\times\,\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)}\big(f_{0,1}(t,\eta)+\mathrm{Bi}_{0,1}T(t,\eta)\big)\,H(t-x,\mathrm{Fo}-\eta)\,dt\,d\eta=\chi v_*(\mathrm{Fo})\,\big(\delta_*(\mathrm{Fo})-A_*a(\mathrm{Fo})\,|x|\big)\,. \tag{25}$$

For the temperature of the bodies, we have the following integral representations:

$$T_1(x,y,\mathrm{Fo})=\frac{1}{\pi}\,\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_{0,1}(t,\eta)\,\Phi_1(t-x,y,\mathrm{Fo}-\eta)\,dt\,d\eta$$

$$+\frac{1}{\pi}\,\mathrm{Bi}_{0,1}\,\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} T(t,\eta)\,\Phi_1(t-x,y,\mathrm{Fo}-\eta)\,dt\,d\eta\,,$$

$$T_2(x,y,\mathrm{Fo})=\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_{0,2}(t,\eta)\,\Phi_2(t,x,y,\mathrm{Fo}-\eta)\,dt\,d\eta\,.$$

But if the surface $y = 0$ outside the contact area is heat-insulated ($\mathrm{Bi}_{0,1} = 0$), then the problem is simplified: one has to solve a system of only two integral equations for the functions $f_{0,j}(x, \mathrm{Fo})$:

$$\frac{1}{\pi}\,\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_{0,1}(t,\eta)\,\Phi_1(t-x,0,\mathrm{Fo}-\eta)\,dt\,d\eta-\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_{0,2}(t,\eta)\,\Phi_2(x,t,0,\mathrm{Fo}-\eta)\,dt\,d\eta=0\,, \tag{26}$$

$$\frac{1}{\pi}\int_{-a(\mathrm{Fo})}^{a(\mathrm{Fo})}\big(f_{0,1}(t,\mathrm{Fo})+\Lambda f_{0,2}(t,\mathrm{Fo})\big)\,\Delta(t-x)\,dt$$

$$-\frac{1}{\pi}\,\chi v_*(\mathrm{Fo})\,\partial_{\mathrm{Fo}}\int_0^{\mathrm{Fo}}\int_{-a(\eta)}^{a(\eta)} f_{0,1}(t,\eta)\,H(t-x,\mathrm{Fo}-\eta)\,dt\,d\eta=\chi v_*(\mathrm{Fo})\,\big(\delta_*(\mathrm{Fo})-A_*a(\mathrm{Fo})\,|x|\big)\,. \tag{27}$$

Moreover, if at the initial instant of time $v_*(0) = 0$, then $f_{0,j}(x, 0) = T(x, 0)$ and we find the contact pressure from the integral equation

$$\frac{1}{\pi}\int\limits_{-a(0)}^{a(0)} p\,(t,0)\,\Delta\,(t-x)\,dt = \delta_*\,(0) - A_* a\,(0)\,\left|x\right| .$$

The problem can be reduced to one integral equation in the case of heat insulation of the die and the layer surface outside the contact area ($\mathrm{Bi}_{0,1} = 0$). Then we get the equation for the relative contact pressure defining the temperature and stressed–strained states of the tribosystem

$$\frac{1}{\pi}\int\limits_{-a(\mathrm{Fo})}^{a(\mathrm{Fo})} p\,(t,\mathrm{Fo})\,\Delta\,(t-x)\,dt - \frac{\chi}{\pi}\,\partial_{\mathrm{Fo}}\int\limits_{0}^{\mathrm{Fo}}\int\limits_{-a(\eta)}^{a(\eta)} v_*\,(\eta)\,p\,(t,\eta)\,H\,(t-x,\mathrm{Fo}-\eta)\,dt\,d\eta = \delta_*\,(\mathrm{Fo}) - A_* a\,(\mathrm{Fo})\,\left|x\right| . \quad (28)$$

For the layer temperature, we have the following integral representation:

$$T_1\,(x,y,\mathrm{Fo}) = \frac{\chi}{\pi}\,\partial_{\mathrm{Fo}}\int\limits_{0}^{\mathrm{Fo}}\int\limits_{-a(\eta)}^{a(\eta)} v_*\,(\eta)\,p\,(t,\eta)\,\Phi_1\,(t-x,y,\mathrm{Fo}-\eta)\,dt\,d\eta . \quad (29)$$

All the above formulas for the kernels of the integral equations hold provided that the corresponding Bi values in them are assumed to be zero.

All systems of integral equations — (19)–(21), (23)–(25), (26)–(27) or (28) — are considered under the condition of changing in x within the boundaries $\left|x\right| \le a(\mathrm{Fo})$ and are closed by the balance condition

$$\int\limits_{-a(\mathrm{Fo})}^{a(\mathrm{Fo})} p\,(x,\mathrm{Fo})\,dx = P_*\,(\mathrm{Fo}) . \quad (30)$$

The unknown half-width of the contact segment $a(\mathrm{Fo})$ is determined from the condition of boundedness of contact stresses $p(\pm a(\mathrm{Fo}), \mathrm{Fo}) = 0$ used only in the case of pressing an unbounded or a high ($b > a(\mathrm{Fo})/\sin(\alpha_0)$) wedge. Otherwise $a(\mathrm{Fo}) = b\sin(\alpha_0)$.

The scheme of solving the system of integral equations is known [8, 9]. But here some points should be made more precise:

1. Asymptotic analysis of the kernels of integral equations permits stating that kernels $H(x, \mathrm{Fo})$, $\Phi_1(x, y, \mathrm{Fo})$, and $\Phi_2(x, t, y, \mathrm{Fo})$ at $y \neq 0$, $\mathrm{Fo} > 0$ will be regular, and kernels $\Delta(x)$, $\Phi_1(x, 0, \mathrm{Fo})$, and $\Phi_2(x, t, 0, \mathrm{Fo})$, when $t \to x$, $\mathrm{Fo} > 0$, have a logarithmic singularity. Then, upon time discretization of the integral equations by the known scheme [8, 9], which, as a consequence of the fulfillment of the conditions $\Phi_1(x, y, 0) = 0$, $\Phi_2(x, t, y, 0)$ and $H(x, 0) = 0$ can justifiably be used, the contact pressure and functions $f_{0,j}(x, \mathrm{Fo})$ at each instant of time $\mathrm{Fo} = \mathrm{Fo}_k$ are given in the following form:

$$p\,(x,\mathrm{Fo}_k) = \frac{2A_*}{\pi}\ln\left(\frac{1+\sqrt{1-x^2}}{\left|x\right|}\right) + \frac{\psi\,(x,\mathrm{Fo}_k)}{\sqrt{1-x^2}}\,,\quad f_{0,j}\,(x,\mathrm{Fo}_k) = \frac{2A_*}{\pi}\ln\left(\frac{1+\sqrt{1-x^2}}{\left|x\right|}\right) + \frac{\varphi_{0,j}\,(x,\mathrm{Fo}_k)}{\sqrt{1-x^2}}\,, \quad (31)$$

where $\psi(x, \mathrm{Fo}_k)$ and $\varphi_{0,j}(x, \mathrm{Fo}_k)$ are continuously differentiable and bounded functions for which the representation is chosen in the form of an interpolation Lagrange polynomial of power n [14] by Chebyshev polynomials of the first kind $T_n(x)$ [10] of order n:

$$\psi\,(x,\mathrm{Fo}_k) = \frac{1}{n}\sum_{i=1}^{n}\psi\,(x_i,\mathrm{Fo}_k)\left(1 + 2\sum_{m=1}^{n-1} T_m\,(x_i)\,T_m\,(x)\right), \quad (32)$$

where $x_i = \cos\left(\frac{2i-1}{2n}\pi\right)$ ($i = 1, ..., n$) are zeros of the Chebyshev polynomial of the first kind of order n, and with the logarithm in formulas (31) the asymptotic value of the contact pressure when $x \to 0$ is given. The expression for the functions $\varphi_{0,j}(x, \mathrm{Fo}_k)$ is analogous. For the temperature of the bodies in the contact region, we choose the representation in the form of a Lagrange interpolation polynomial of the form (32), since from the physical considerations it is clear that with any relations between the thermophysical characteristics of the bodies it is a continuous and bounded function.

2. Having given the value of the half-width of the contact segment $a(\mathrm{Fo}_k)$, we choose the value of the die upsetting $\delta_*(\mathrm{Fo}_k)$ so that the contact pressure satisfies the balance condition. In addition, the conditions $\psi(\pm 1, \mathrm{Fo}_k) > 0$ (at nonideal thermal contact) or $\varphi_{0,1}(\pm 1, \mathrm{Fo}_k) + \Lambda\varphi_{0,2}$ (± 1, $\mathrm{Fo}_k > 0$) (at ideal contact) must be fulfilled when it is obvious that these conditions can only be met by pressing-in a short wedge with an insignificant pressing-in force $P_*(\mathrm{Fo})$. But if an unbounded or a high wedge is pressed in, these conditions will inevitably be violated. Choosing the boundaries of the contact region, we attain fulfillment of the approximate conditions that are due to the numerical approach to the solution of the system: $|\psi(\pm 1, \mathrm{Fo}_k)| < \varepsilon$ or $|\varphi_{0,1}(\pm 1, \mathrm{Fo}_k) + \Lambda\varphi_{0,2}(\pm 1, \mathrm{Fo}_k)| < \varepsilon$, where ε is some number defining the computational error (as a rule, $\varepsilon = 10^{-5}$). On the basis of the theorems from [7], the fulfillment of the last two conditions is equivalent to the fact that

$$p\,(x, \mathrm{Fo}_k) = \frac{2A_*}{\pi}\ln\left(\frac{1+\sqrt{1-x^2}}{|x|}\right) + \psi_1\,(x, \mathrm{Fo}_k)\sqrt{1-x^2}\,, \tag{33}$$

where $\psi(x, \mathrm{Fo}_k)$ is a continuously differentiable and bounded function for which, as for (32), an interpolation Lagrange polynomial of power n is constructed [14]:

$$\psi_1\,(x, \mathrm{Fo}_k) = \frac{2}{n+1}\sum_{i=1}^{n}\psi_1\,(x_i, \mathrm{Fo}_k)\left(1-x_i^2\right)\left(1+\sum_{m=1}^{n-1}U_m\,(x_i)\,U_m\,(x)\right) \tag{34}$$

by Chebyshev polynomials of the second kind $U_n(x)$ [10] of order n, $x_i = \cos\left(\frac{i}{n+1}\pi\right)$ ($i = 1, ..., n$) denotes zeros of the Chebyshev polynomial of the second kind of order n. Then the final expression for the temperature of the bodies in the contact region is also given in the form of a polynomial of form (34).

The use of formulas (33), as well as of the expressions for the functions $f_{0,j}$ and the temperature of the contact segment in terms of interpolation Lagrange polynomials by Chebyshev polynomials, makes it possible to determine the real distribution of the contact pressure and contact temperature at nonlinear values of $\delta_*(\mathrm{Fo}_k)$ and $a(\mathrm{Fo}_k)$. For calculations, it is enough to take a time-discretization step $\mathrm{Fo}_1 = 0.05$ and a power of the Lagrange interpolation polynomials $n = 21$. Then the relative computational error does not exceed 5%.

3. The discontinuity of the slope of the tangent to the profile inside the contact space (the point $x = 0$) is responsible for the existence of the logarithmic singularity of the contact pressure, and if a low wedge is pressed in, then $p(x, \mathrm{Fo})$ will also have a root singularity on the edge of the interaction region.

4. Due to the heat insulation of the side surface of the wedge outside the interaction area there is no stationary temperature distribution in the wedge-shaped region in this formulation of the problem for both the unbounded and the bounded body. This conclusion follows from the analysis of the kernel of the integral representation of the wedge temperature (17) and is confirmed by the further numerical calculations. As a consequence, even in the case of the existence of asymptotic (stationary) values of the functions of the pressing force $P_*(\mathrm{Fo})$ and the velocity of displacement $v_*(\mathrm{Fo})$ in the tribosystem under consideration, the stationary distribution of the temperature fields and thermoelastic stresses and displacements will not reach the steady state. The steady state can only be reached in a tribosystem consisting of a heat-conducting layer and a heat-insulated die for which the integral equation (28) has been constructed. Evidently, in this tribosystem the existence of the steady state is possible only in the case where asymptotic (stationary) values of the functions $P_*(\mathrm{Fo})$ and $v_*(\mathrm{Fo})$ will exist.

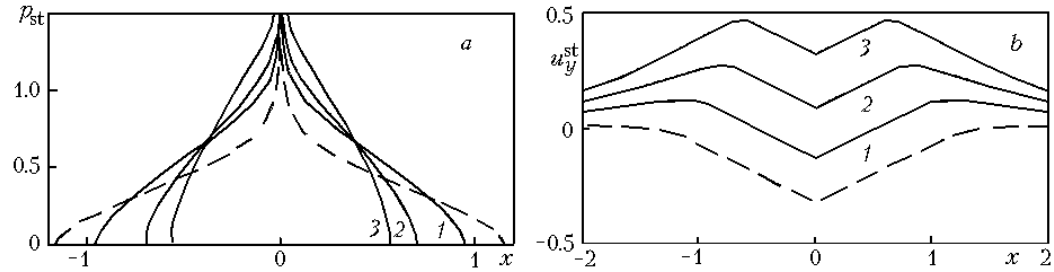

Fig. 2. Distribution of the contact pressure (a) and normal displacements of the contact surface of the layer $u_y(x,\ 0)$ (b) of the stationary problem for a heat-insulated die ($v = 0.3$, $Bi_{0,1} = 0$, $Bi_{1,1} = 2.0$, $\alpha_0 = \pi/4$, $(Eh)/(2P_0(1-\nu^2)) = 0.25$, $\chi = 0.5$ (1), 1.0 (2), and 1.5 (3)). The dashed curve corresponds to the contact pressure of the force problem ($\chi = 0$).

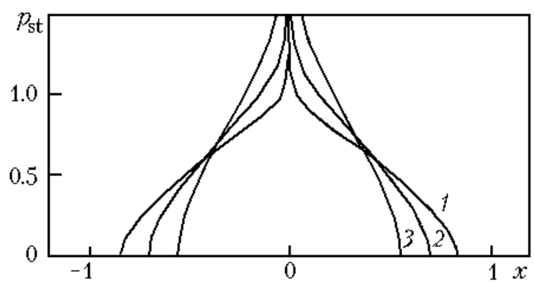

Fig. 3. Contact pressure distributions of the thermoelastic stationary problem at various values of the apex angle of the wedge ($\chi = 1.0$, $\alpha_0 = \pi/3$ (1), $\pi/4$ (2), and $\pi/6$ (3)).

Analysis of the Results. As was shown in [3], the contact pressure distribution in the stationary problem and, as a consequence, mechanical and thermophysical characteristics of the tribosystem, such as thermoelastic displacements, stresses, and temperature, depend on the value of the parameter χ defining the heat-release intensity. The following effects of the behavior of the considered tribosystem take place:

(1) an increase in χ causes a decrease in the upsetting and in the value of the contact segment (Fig. 2), which is a consequence of the surface bulging $y = 0$;

(2) balance of the die is possible at negative values of δ_* when the increment of the pressing force intensity P_0 additionally warps the contact surface;

(3) at fixed χ an increase in the pressing-force intensity P_0 increases $|\delta_*|$ and the value of the contact-segment half-width a.

Prevalence of any one of the types of external action — purely mechanical or thermal — is not characteristic of this tribosystem, which shows up as the absence of critical values of the contact segment half-width [8] at which a change in the pressing force does not influence the size of the interaction space. Numerical calculations of the stationary thermoelastic problem show the simultaneous action of the thermomechanical factors:

1. As the apex angle α_0 of the die increases, there is an increase in the contact-segment half-width in the thermoelastic problem as well.

2. Unlike the pure elastic problem, where an increase in the pressing force causes an increase in the upsetting, in the thermoelastic problem a larger contact space enhances the heat generation and causes a warpage of the layer surface such that the negative value of δ_* increases.

These conclusions are illustrated in Fig. 3, where at $\alpha_0 = \pi/3$ $\delta_* \approx -0.1305$, at $\alpha_0 = \pi/4$ $\delta_* \approx -0.09$, and in the third case $\delta_* \approx -0.027$ ($\alpha_0 = \pi/6$).

Analysis of the contact pressure distribution at a nonstationary heat release caused by the pressing force $P_*(\text{Fo})$ and the velocity of motion $v_*(\text{Fo})$ varying according to the laws (1) $P_*(\text{Fo}) = 1 - \exp(-\text{Fo})$, $v_*(\text{Fo}) = 1$ (Fig. 4a) and (2) $P_*(\text{Fo}) = 1$, $v_*(\text{Fo}) = 1 - \exp(-\text{Fo})$ (Fig. 4b) shows that with the first law of change with time in the load and

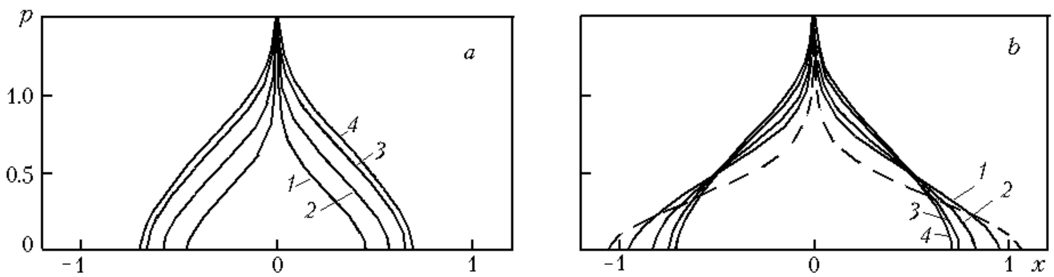

Fig. 4. Contact pressure distributions under nonstationary heat generation caused by the pressing force P_*(Fo) and the velocity of motion v_*(Fo) for some values of the dimensionless time of Fo [1) 0.5, 2) 1.0, 3) 2.0, and 4) 4.0) when $\chi = 1.0$. The dashed curve corresponds to the pressure in the stationary problem.

velocity of motion elastic deformations prevail and with the second law thermal deformations prevail. The duration of the transient process for the contact pressure is longer (about 6Fo) than the time of attaining a stationary value of the pressing load or the velocity (4.5Fo).

Conclusions. Note that in the case of pressing-in a short wedge, the existence of a solution with a root peculiarity of contact stresses on the edge of the contact region is possible. However, an increase in the heat-generation intensity causes a bulge in the contact surface of the layer, which leads to a decrease in the contact segment and a regularization of the contact stresses (they vanish at the ends of the contact region), i.e., the die on the edge of the contact space begins to separate from the layer surface. However, in both the elastic and the thermoelastic problem the contact pressure preserves the logarithmic singularity at the discontinuity point of the tangent $x = 0$ to the die surface.

NOTATION

A_*, dimensionless parameter of the wedge geometry; a(Fo), dimensionless half-width of the interaction area; $a(\tau)$, time-dependent half-width of the contact area, m; $\mathrm{Bi}_{0,1}$, $\mathrm{Bi}_{0,2}$, $\mathrm{Bi}_{1,1}$, Biot criteria; b, radius of the circular arc of the wedge, m; E, Young modulus, N/m^2; erfc (x), error function; exp (x), exponential function; Fo, Fourier criterion; Fo_k, discrete value of time in the numerical solution of the integral equation; $F(x, y, \tau)$, thermoelastic potential of displacements, m^2; $f_j(x, \tau)$, function of the combination of the temperature and thermal flow on the contact surface, K/m; $f_{0,j}(x, \mathrm{Fo})$, dimensionless function of the combination of the temperature and thermal flow on the contact surface; f, friction coefficient; $H(x, \mathrm{Fo})$, kernel of the integral equation; h, layer thickness, m; h_0, heat conductivity on the contact surface, W/(m^2·K); h_*, dimensionless heat conductivity of the contact surface; $J_n(x)$, Bessel function of the first kind of order n; $K_1(\varphi, n)$, kernel of the finite integral Fourier transform; $K_2(\rho, \mu_{m,n})$, kernel of the finite integral Hankel transform; k, ratio of the thermal diffusivities of the bodies; k_j, thermal diffusivity, m^2/sec; $P(\tau)$, pressing force, N; P_0, intensity of the pressing force, N; P_*(Fo), dimensionless function of the pressing force; $p(x, \tau)$, contact pressure, N/m^2; $p(x, \mathrm{Fo})$, dimensionless function of the contact pressure; p_{st}, dimensionless function of the contact pressure of the stationary problem; $S(x)$, Heaviside function; t, integration variable; T_j, temperature, K; $T_{0,j}(x, \mathrm{Fo})$, $T(x, \mathrm{Fo})$, dimensionless functions of the contact segment temperature; $T_n(x)$, Chebyshev polynomial of the first kind of order n; $U_n(x)$, Chebyshev polynomial of the second kind of order n; u_x, u_y, components of the displacement vector, m; u_y^{st}, dimensionless function of normal displacements of the contact surface of the layer of the stationary problem; $v(\tau)$, velocity of motion, m/sec^2; v_0, scale of change in the velocity, m/sec^2; v_*(Fo), dimensionless velocity function; x, y, z, Cartesian coordinates, m; x_i, zeros of Chebyshev polynomials; α, linear thermal expansion coefficient, K^{-1}; α_0, half-opening of the die, rad; β_0, γ_0, γ_1, coefficients of heat exchange between the corresponding planes of the interacting bodies and the environment, m^{-1}; $\Delta(x)$, kernel of the integral equation for determining the contact pressure; $\delta(\tau)$, upsetting of the die, m; δ_*(Fo), dimensionless function of the die upsetting; ε, computational error; ζ, η, integration variables; Λ, ratio of heat conductivities of the bodies; λ_j, heat-conductivity coefficient, W/(m·K); μ, nonnegative roots of the transcendental equations of the Sturm–Liouville problem; ν, Poisson coefficient; ξ, parameter of the integral Fourier transform; ρ, φ, polar coordinates, m, rad; σ_x, σ_y, τ_{yx}, τ_{yz}, components of the stress tensor, N/m^2; τ, time, sec; Φ_j, kernel of the integral representation of the temperature; χ, parameter defining the heat-generation intensity; $\psi(x, \mathrm{Fo}_k)$, $\psi_1(x, \mathrm{Fo}_k)$, $\varphi_{0,j}(x, \mathrm{Fo}_k)$, Lagrange interpolation polynomials. Subscripts: i, j, indices of summing and defining the temperature of the corresponding body; k, summing index and discrete value of Fo; m, n, summing indices and indices for selecting eigenvalues of the Sturm–Liouville problem; st, stationary solution of the problem; x, y, z, components of the vector of displacements and the tensor of stresses in the direction of the corresponding Cartesian coordinates; 0, dimensional parameters of the problem and particular representations of the problem solution functions; 1, particular representations of the problem solution functions; *, dimensionless parameters of the problem.

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